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D-ll Physics Hons.

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Maxwell's equations for static electric & magnetic field

We have ~~encounterd~~ encountered the following laws, specifying the div. and curl of electric & magnetic fields —

	differential form	Integral form
①	$\vec{\nabla}\cdot\vec{D} = \rho$	$\oint \vec{D}\cdot d\vec{s} = \int \rho\, dv$ → Gauss law
②	$\vec{\nabla}\cdot\vec{B} = 0$	$\oint \vec{B}\cdot d\vec{s} = 0$ (non existence of magnetic poles)
③	$\vec{\nabla}\times\vec{E} = 0$	$\oint \vec{E}\cdot d\vec{l} = 0$ (Conservative nature of electrostatic field)
④	$\vec{\nabla}\times\vec{H} = \vec{J}$	$\oint \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s}$ (Ampere's law)

These eqn represent the state of e.m. Theory over a century ago, when maxwell began his work. then Maxwell given four eqs., which are same as above eqs. —

① $\vec{\nabla}\cdot\vec{D} = \rho \Rightarrow \boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}}$ $\qquad \oint \vec{D}\cdot d\vec{s} = \int \rho\, dv$ (Gauss's law)

② $\boxed{\vec{\nabla}\cdot\vec{B} = 0} \Rightarrow$ $\qquad \oint \vec{B}\cdot d\vec{s} = 0$

③ $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$ $\qquad \oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{s}$ (Faraday's law) -

④ $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ $\qquad \oint \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) d\vec{s}$ (Ampere's law)

or, $\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\cdot\frac{\partial\vec{D}}{\partial t}}$

above, four eqs. are Maxwell's equation for Time varying field

$(\vec{D} = \varepsilon_0\vec{E})$ —

$(\vec{B} = \mu_0\vec{H})$ —

Maxwell's Displacement Current:-

For, Ampere's law we have:

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}} \quad \text{——— (1)}$$

div of both sides

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J})$$

$$0 = \mu_0(\vec{\nabla}\cdot\vec{J})$$

put in above eqn

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \quad \text{——— (A)}$$

$$0 = \mu_0\left(-\frac{\partial\rho}{\partial t}\right)$$

or, $\frac{\partial\rho}{\partial t} = 0$, which is not possible,

the eqn of Continuity requires that $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t} \neq 0$

this implies that Ampere's law only holds for steady current where the rate of flow of charge is const.

let us, extend eqn (1) for time varying condns by adding a term to R.H.S. of eqn (1)

$$\vec{\nabla}\times\vec{B} = \mu_0(\vec{J}+\vec{J_D}) \quad \text{——— (2)}$$

on taking, again div. of both sides

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J}+\vec{\nabla}\cdot\vec{J_D}) \qquad -\frac{\partial\rho}{\partial t}$$

$$0 = \mu_0(\vec{\nabla}\cdot\vec{J}+\vec{\nabla}\cdot\vec{J_D})$$

or, $\vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{J_D}$

$$\vec{\nabla}\cdot\vec{J_D} = \frac{\partial\rho}{\partial t} \quad \text{——— (3)}$$

from Gauss's law:-

we have

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

differentiate w.r. to time

$$\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) = \vec{\nabla}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{1}{\varepsilon_0}\frac{\partial\rho}{\partial t}$$

So, $\vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0} \cdot \frac{\partial \rho}{\partial t}$

$\left\{ \because \vec{\nabla} \cdot \vec{J_D} = \frac{\partial \rho}{\partial t} \right\}$

or, $\vec{\nabla} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0} \cdot (\vec{\nabla} \cdot \vec{J_D})$

$\vec{\nabla} \cdot \left(\frac{\partial \vec{E}}{\partial t} - \frac{1}{\varepsilon_0} \cdot \vec{J_D} \right) = 0$

or, $\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon_0} \cdot \vec{J_D}$

$\vec{J_D} = \varepsilon_0 \cdot \frac{\partial \vec{E}}{\partial t}$ $\{ \because \vec{D} = \varepsilon_0 \vec{E} \}$

$$\boxed{\vec{J_D} = \frac{\partial \vec{D}}{\partial t}}$$

put this relation in (2), then Ampere's law became

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \frac{\partial \vec{D}}{\partial t}} \quad \text{——— (4)}$$

Still the same, experimentally varified, law for steady state phenomena, but now mathematically consistent with the continuity eq^n (A) for time dependent fields. Maxwell called the added term in eq^n (2) $\vec{J_D} = \frac{\partial \vec{D}}{\partial t}$ 'displacement current'. It represents that a changing electric field causes a magnetic field.

eq^n (4) can be written as,

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$(\vec{B} = \mu_0 \vec{H})$

This is known as the Ampere's law for time dependent fields.